Scale-uniform quantitative unique continuation principle

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For many elliptic partial differential equations it is known that a solution cannot vanish of arbitrary order unless it is identically zero. This can be proven using a Carleman estimate. The latter is also useful to prove a quantitative version of the unique continuation principle.

We are interested in a particular quantitative and scale free version of this result: Consider an eigenfunction ψ of a Schroedinger equation on a cube Λ of size L with periodic boundary conditions. Assume that the L^2-norm of the eigenfunction is one.

The cube of size L can be decomposed into unit cubes. Place in each unit cube arbitrarily a ball with small, but fixed radius. We derive a lower bound on the L^2-norm of the eigenfunction when integrated over the union of the small balls. The bound is independent of the size L, and depends in an explicit way on the other parameters entering the problem.

$$\int_{\bigcup \bullet} |\psi|^2 \geq C \int_{\Lambda} |\psi|^2$$